The study carried out showed that with reaction of converging conical SW in porous specimens of aluminum, magnesium, graphite, and mixtures of aluminum with iron and aluminum with potassium iodide, a Mach reaction is observed. In the test arrangement used with porous iron specimens no stationary Mach disk was established, and apparently this is connected with the insufficient height of the shell.

In the case of Mach reaction there is an increase in pressure transmitted to the plexiglas barrier with an increase in specimen porosity and a reduction in Mach disk size.

LITERATURE CITED

1. J. Thouvenin, "Effect of a shock wave on a porous solid," in: IVth Symposium (internat.) on Detonation, Washington (1967).
2. F. A. Baum, L. P. Orlenko, K. P. Stanyukovich, et al., Physics of Blasting, second edition [in Russian], Nauka, Moscow (1975).
3. A. N. Dremin and I. A. Karpukhin, "Method for determining shock adiabatic curves for dispersed substances," Prikl. Matem. i Tekh. Fiz., No. 3 (1960).

PROPAGATION OF A SPHERICAL WAVE IN NONLINEARLY COMPRESSIBLE AND ELASTOPLASTIC MATERIALS

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The problem of spherical wave propagation in soil under the action of an intense uniformly decreasing load $\sigma_0(t)$ applied to the boundary of a cavity with radius r_0 is considered. Soil with a high stress level is modeled either by ideally nonlinearly compressible or elastoplastic material, taking account of linear irreversible unloading for the material. In contrast to [1-7], in order to describe material movement use is made of strain theory [8] with determining functions $\sigma = \sigma(\varepsilon)$, $\sigma_i = \sigma_i(\varepsilon_i)$, where ε, ε_i, σ, σ_i are the first and second invariants of strain and stress tensors. During material loading these functions are presented in the form of polynomials

$$\sigma(\varepsilon) = (\alpha_1 + \alpha_2|\varepsilon|)\varepsilon,\ \sigma_i\varepsilon_i) = (\beta_1 - \beta_2\varepsilon_i)\varepsilon_i,$$

in which constant coefficients α_i, $\beta_i (i = 1, 2)$ are determined by experiment, taking account of the triaxial stressed state of soil. Solution of the problem is constructed by an analytically reversible method, with prescribed shape for the shock-wave (SW) surface in the form of a second-degree polynomial relating to time t and a numerical method of characteristics for a prescribed arbitrarily decreasing load $\sigma_0(t)$. On the basis of the analytical equations obtained, calculations are carried out for material parameters (including loading profile) in a computer and stresses and mass velocity of plastic and elastoplastic materials are compared.

This work is a continuation of [9, 10] for studying the characteristic features of spherical wave propagation in soils and the behavior of its parameters with intense effects.

1. Let at the boundary of a spherical cavity $r = r_0$ a uniformly decreasing load $\sigma_0(t)$ be applied. In the case of considering the problem within the limits of a nonlinearly compressible material with fulfillment of the first expression (0.1) taking account of $\sigma(\varepsilon) = \sigma_{rr} = \sigma_{\varphi\varphi} = \sigma_{\theta\theta} = -p$, $\varepsilon = (1 - \rho_0/\rho) > 0$ (p is pressure, ρ_0 is initial material density), a spherical SW $r = R(t)$ will propagate in the soil, at whose front the soil is instantaneously loaded in a nonlinear fashion while beyond it in the region of disturbance there is

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material unloading. Then from equations of motion, material continuity and state [9], we have an equation relating to mass velocity $u(r, t)$

$$\frac{\partial^2 u}{\partial t^2} - c_p^2 \frac{\partial^2 u}{\partial r^2} - 2\frac{c_p^2}{r}\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right) = 0,$$

taking account of the relationship at the front $r = R(t)$ and the first equality (0.1) with prescribed $\dot{R}(t) = dR/dt$ (in this case all of the material parameters at the SW front are known time functions) is resolved in the form

$$u(r, t) = \frac{1}{r}\left\{\int_{r_0}^{r-c_pt} d\xi_2 \int_{r_0}^{\xi_2} \Phi_1(\xi_1)\, d\xi_1 - \int_{r_0}^{r+c_pt} d\xi_2 \int_{r_0}^{R[F(\xi_2)]-c_pF(\xi_2)} \Phi_1(\xi_1) d\xi_1 - \right.$$
$$\left. - \frac{\rho_0}{\alpha_2}\int_{r_0}^{r+c_pt} \frac{\dot{R}[F(\xi_2)]\ddot{R}[F(\xi_2)]R[F(\xi_2)]}{\Delta_2[F(\xi_2)]}\, d\xi_2\right\} - \frac{1}{r^2}\left\{\int_{r_0}^{r-c_pt} d\xi_3 \int_{r_0}^{\xi_3} d\xi_2 \int_{r_0}^{\xi_2} \Phi_1(\xi_1) d\xi_1 - \right.$$
$$- \int_{r_0}^{r+c_pt} d\xi_3 \int_{r_0}^{\xi_3} d\xi_2 \int_{r_0}^{R[F(\xi_2)]-c_pF(\xi_2)} \Phi_1(\xi_1) d\xi_1 -$$
$$\left. - \frac{\rho_0}{\alpha_2}\int_{r_0}^{r+c_pt} d\xi_3 \int_{r_0}^{\xi_3} \frac{\dot{R}[F(\xi_2)]\ddot{R}[F(\xi_2)]R[F(\xi_2)]}{\Delta_2[F(\xi_2)]}\, d\xi_2 + m_1c_pt + n_1\right\}, \tag{1.1}$$

$$\Delta_2(t) = \sqrt{\frac{(1-\alpha_1/\alpha_2)^2}{4} - \frac{\rho_0\dot{R}^2(t) - \alpha_1}{\alpha_2}}, \quad c_p = \sqrt{E/\rho_0},$$

where E is the tangent of the slope of the branch of the unloading diagram p—ε with axis Oε; c_p is the disturbance propagation velocity in the unloading region; m_1 and n_1 are constant coefficients determined from the condition at the wave front with t = 0; $\Phi_1(\xi)$ is a known function depending on material parameters at the SW front; $F(z_{1,2})$ is the root of the equation $R(t) + c_pt = z_{1,2}$ relating to t.

Taking account of the boundary condition for the problem $p(r_0, t) = \sigma_0(t)$, we integrate the equation of material movement [9] for r from $r = r_0$ to $r = R(t)$. Then in order to determine the load $\sigma_0(t)$ taking account of (1.1)

$$\sigma_0(t) = p^*(t) + \rho_0 \int_{r_0}^{R(t)} \frac{\partial u}{\partial t}(r, t)\, dr,$$

where $p^*(t)$ is material pressure at the front $r = R(t)$ which with precribed $R(t)$ is assumed to be a known value. It is noted that in this case the rule for material unloading

$$p(r, t) = p^*(t) = E(\varepsilon - \varepsilon^*)$$

makes it possible to determine volumetric strain $\varepsilon(r, t)$.

2. We solve the problem within the limits of elastoplastic strains using strain theory for soil plasticity [8]. Then a study of the material movement equation

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{rr}}{\partial r} + 2\frac{(\sigma_{rr} - \sigma_{\varphi\varphi})}{r} \tag{2.1}$$

taking account of the relationship between stress and strain components

$$\sigma_{rr} = \lambda\varepsilon + 2G\varepsilon_{rr}, \quad \sigma_{\varphi\varphi} = \sigma_{\theta\theta} = \lambda\varepsilon,$$
$$\varepsilon = \partial u/\partial r + 2u/r, \quad \lambda = \sigma/\varepsilon - (2/9)\sigma_i/\varepsilon_i, \quad G = (1/3)\sigma_i/\varepsilon_i \tag{2.2}$$

and (0.1) indicates that with $\alpha_2 - (8/27)\beta_2 > 0$ an SW $r = R(t)$ propagates in the soil, but with $\alpha_2 - (8/27)\beta_2 < 0$ Riemann centered loading waves, which from above intercept the unloading wave $t = f(r)$, are boundaries for the region of material loading and unloading.

Since a spherical SW is a loading—unloading wave and behind its front there is material unloading, then from (2.1), taking account of the Ilyushin unloading theorem expressed by the equation

$$\sigma_{rr}=\sigma_{rr}^{*}(r)+\lambda_0(\varepsilon-\varepsilon^{*}(r))+2G_0(\varepsilon_{rr}-\varepsilon_{rr}^{*}(r)),$$
$$\sigma_{\varphi\varphi}=\sigma_{\varphi\varphi}^{*}(r)+\lambda_0(\varepsilon-\varepsilon^{*}(r))+2G_0(\varepsilon_{\varphi\varphi}-\varepsilon^{*}(r)), \quad (2.3)$$

we obtain

$$\frac{\partial^2 u}{\partial t^2}=a_0^2\left[\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\left(\frac{\partial u}{\partial r}-\frac{u}{r}\right)+\frac{Q(r)}{\rho_0 a_0^2}\right]; \quad (2.4)$$

$$Q(r)=\frac{\partial}{\partial r}\left[\sigma_{rr}^{*}(r)-\rho_0 a_0^2\varepsilon^{*}(r)\right]+\frac{2}{r}\left[\sigma_{rr}^{*}(r)-\sigma_{\varphi\varphi}^{*}(r)-2G_0\varepsilon^{*}(r)\right], \quad (2.5)$$
$$\rho_0 a_0^2=\lambda_0+2G_0,\ \lambda_0=E_1-\frac{2}{9}E_2,\ G_0=\frac{1}{3}E_2,$$

where E_1 and E_2 are tangents for the slope of unloading branches respectively for the $\sigma(\varepsilon)$ and $\sigma_i(\varepsilon_i)$ diagrams with axes ε and ε_i; u is displacement; material parameters relating to the front are labeled with an upper asterisk.

We now consider the case of $\alpha_2-(8.27)\beta_2>0$ with conditions at the wave front and cavity boundary:

$$\sigma_{rr}^{*}=-\rho_0\dot{R}(t)u_t^{*},\ u_t^{*}=-\dot{R}(t)\varepsilon_{rr}^{*},\ u^{*}(r,t)=0 \text{ при } r=R(t); \quad (2.6)$$

$$\sigma_{rr}=-\sigma_0(t) \text{ при } r=r_0,\ t\geqslant 0\ (u_t=\partial u/\partial t=u,\ \dot{R}(t)=\partial R/\partial t). \quad (2.7)$$

Solution of the problem is built up by the reverse method. Let $\dot{R}(t)$ be given, then (2.6) taking account of (2.2) with r = R(t) takes the form

$$\varepsilon^{*}(t)=\varepsilon_{rr}^{*}(t)=-\frac{\rho_0\dot{R}^2(t)-\left(\alpha_1+\frac{4}{9}\beta_1\right)}{\left(\alpha_2-\frac{8}{27}\beta_2\right)},\ u_t^{*}=-\dot{R}(t)\varepsilon^{*}(t). \quad (2.8)$$

By using the d'Alembert equation we obtain a solution for Eq. (2.4)

$$u(r,t)=\frac{\psi'(r-a_0t)+\Phi(r+a_0t)}{r}-\frac{\psi(r-a_0t)+\Phi(r+a_0t)}{r^2}-$$
$$-\frac{r}{3(\lambda_0+2G_0)}\int_{r_0}^{r}Q(r)\,dr+\frac{1}{3(\lambda_0+2G_0)r^2}\int_{r_0}^{r}Q(r)r^3dr \quad (2.9)$$

(ψ and Φ are unknown functions).

By substituting (2.9) in (2.8) we obtain expressions for determining the functions ψ and Φ, which are reproduced due to their bulk. After this, solution of the problem relating to displacements u(r, t) is written as

$$u(r,t)=\frac{1}{r}\left\{\int_{r_0}^{r-a_0t}d\xi_2\int_{r_0}^{\xi_2}\Phi_2(\xi_1)\,d\xi_1-\right. \quad (2.10)$$
$$-\int_{r_0}^{r+a_0t}d\xi_2\int_{r_0}^{R(F_2(\xi_2))-a_0F_2(\xi_2)}\Phi_2(\xi_1)\,d\xi_1+\int_{r_0}^{r+a_0t}\frac{R(F_2(\xi_2))}{(\lambda_0+2G_0)}d\xi_2\int_{r_0}^{R(F_2(\xi_2))}Q(\xi_1)\,d\xi_1-$$
$$\left.-\int_{r_0}^{r+a_0t}\frac{R(F_2(\xi_2))\left[\rho_0\dot{R}^2(F_2(\xi_2))-\left(\alpha_1+\frac{4}{9}\beta_1\right)\right]}{\left(\alpha_2-\frac{8}{27}\beta_2\right)}d\xi_2\right\}-$$
$$-\frac{1}{r^2}\left\{\int_{r_0}^{r-a_0t}d\xi_3\int_{r_0}^{\xi_3}d\xi_2\int_{r_0}^{\xi_2}\Phi_2(\xi_1)\,d\xi_1-\frac{a_0r_0^2\left(1+\dot{R}(0)/a_0\right)}{(\alpha_2-(8/27)\beta_2)}\left[\rho_0\dot{R}^2(0)-\right.\right.$$

$$-\left(\alpha_1 + \frac{4}{9}\beta_1\right)\Big] t - \int_{r_0}^{r+a_0t} d\xi_3 \int_{r_0}^{\xi_3} d\xi_2 \int_{r_0}^{R(F_2(\xi_2))-a_0F_2(\xi_2)} \Phi_2(\xi_1)\, d\xi_1 - \tag{2.10}$$

$$- \int_{r_0}^{r+a_0t} d\xi_3 \int_{r_0}^{\xi_3} \frac{R(F_2(\xi_2))\left[\rho_0 \dot{R}^2(F_2(\xi_2)) - \left(\alpha_1 + \frac{4}{9}\beta_1\right)\right]}{\left(\alpha_2 - \frac{8}{27}\beta_2\right)} \times$$

$$\times d\xi_2 + \int_{r_0}^{r+a_0t} d\xi_3 \int_{r_0}^{\xi_3} \frac{R(F_2(\xi_2))}{(\lambda_0 + 2G_0)} d\xi_2 \int_{r_0}^{R(F_2(\xi_2))} Q(\xi_1)\, d\xi_1 \Bigg\} -$$

$$- \frac{r}{3(\lambda_0 + 2G_0)} \int_{r_0}^{r} Q(r)' d\xi + \frac{1}{3(\lambda_0 + 2G_0) r^2} \int_{r_0}^{r} Q(\xi)\, \xi^3 d\xi,$$

where $F_i(z_i)$ ($i = 1, 2$) is the root of equations $R(t) \mp a_0 t = z_i$ relating to t; $\Phi_2(z)$ is a known function of its argument which is expressed in terms of material parameters at the SW front.

By differentiating (2.10) for t and r we determine mass velocity $u_t(r, t)$ and strain $\varepsilon(r, t)$, and then on the basis of Eq. (2.3) stress components σ_{rr}, and $\sigma_{\phi\phi}$ are calculated. Furthermore, by substituting (2.3), taking account of (2.10) in (2.7), we find the loading profile $\sigma_0(t)$.

3. In order to solve the spherical elastoplastic problem by the characteristic method, introducing the notation

$$u_t = \partial u/\partial t, \quad \varepsilon_{rr}' = \partial u/\partial r$$

from (2.4) we obtain a system

$$\frac{\partial u_t}{\partial t} = a_0^2 \left[\frac{\partial \varepsilon_{rr}}{\partial r} + \frac{2}{r}\left(\varepsilon_{rr} - \frac{u}{r}\right) + \frac{Q(r)}{\rho_0 a_0^2}\right], \quad \frac{\partial u_t}{\partial r} = \frac{\partial \varepsilon_{rr}}{\partial t}. \tag{3.1}$$

If it is considered that at the front $r = R(t)$ the displacement $u(r, t) = 0$, then the system of Eqs. (3.1) permits the following equations for characteristics and ratios in them:

$$r_{1,2}' = \mp a_0, \quad du = u_t dt + \varepsilon_{rr} dr, \tag{3.2}$$

$$du_t - a_0^2 \left[\frac{2}{r}\left(\varepsilon_{rr} - \frac{u}{r}\right) + \frac{Q(r)}{\rho_0 a_0^2}\right] dt - r_{1,2}' d\varepsilon_{rr} = 0.$$

On the basis of (3.2) and taking account of (2.2), (2.3), (2.6) and (2.7) a scheme for working out the problem similar to the numerical scheme in [11] is developed, and calculations were carried out in a computer for a specific soil structure.

4. Calculations were carried out for the case when the shape of the SW surface was given as a second-degree polynomials $R(t) = r_0 + R_1 t - (R_2/2)t^2$, $\dot{R}(t) > 0$ with initial soil parameters: $\rho_0 = 0.02 \cdot 10^4$ kg·sec²/m⁴, $\sigma_0(0) = 105 \cdot 10^4$ kg/m², $r_0 = 1$ m, $R_2 = 2 \cdot 10^2 R_1$, $\alpha_1 = 12.127 \cdot 10^4$ kg/m², $\alpha_2 = 58.73 \cdot 10^7$ kg/m², $\beta_1 = 35.83 \cdot 10^6$ kg/m², $\beta_2 = 11.64 \cdot 10^8$ kg/m², $E_1 = 14 \cdot 10^7$ kg/m², $E_2 = 2 \cdot 10$ kg/m².

Results of the calculations in a computer are presented in Figs. 1-4 for stresses, mass velocity, and loads in relation to time and spatial coordinate r, and also at the SW front $r = R(t)$. Here broken and solid lines correspond to ideal lnearly compressible and elastoplastic materials. Calculations with $t = 0$ taking account of relationship (2.6) in the case of fulfilling the boundary condition $\sigma_{rr}(r_0, 0) = -p(r_0, 0) = -\sigma_0(0) = -105$ kg/cm² indicate that the SW velocity $\dot{R}(0) = R_1$ is different for linearly compressible and elastoplastic materials, and it is $R_1 = 391$ and 420 m/sec.

Consequently, the reason for elastoplastic material disturbance becomes more extensive than the reason for disturbance of an ideal nonlinearly compressible material.

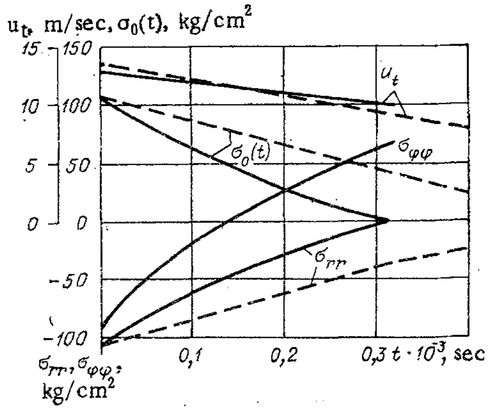

Fig. 1

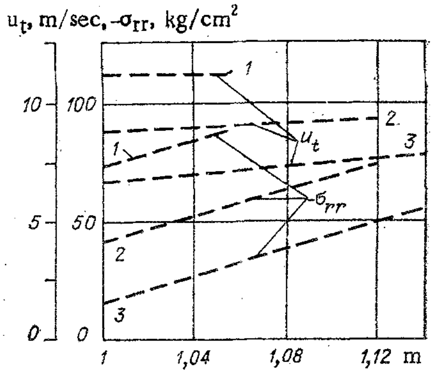

Fig. 2

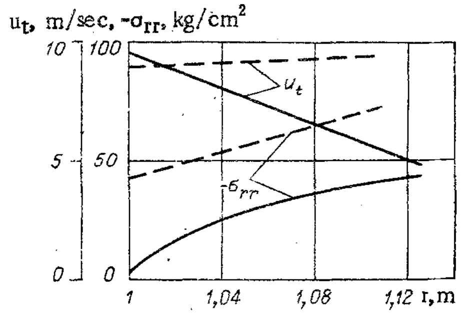

Fig. 3

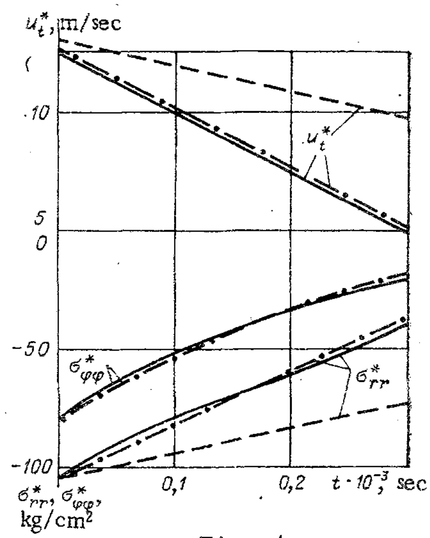

Fig. 4

It can be seen from Fig. 1 ($r = r_0 = 1$) that in the case of modeling soil by a nonlinearly compressible material compared with the elastoplastic problem, solutions found by the reverse method for loading profile $\sigma_0(t)$ vary comparatively slowly with time, and with $t > 0$ the absolute value is of greatest importance. This is explained by the fact that in considering the problem within the limits of a nonlinearly compressible material the soil is compressed by the same pressure from all sides, whereas in using dynamic plasticity theory $\sigma_{rr} > \sigma_{\phi\phi} = \sigma_{\theta\theta}$ and the damping process for $\sigma_0(t)$ in the latter proceeds comparatively rapidly. In addition, all of the material parameter with $r = r_0$ in relation to t have a damping character.

A similar rule for the change in parameters $\sigma_{rr} = -p$, $\sigma_{\phi\phi}$, $u_t = u$ with time is observed with $r > r_0$. However, with $r > r_0$ the intensity of the above-mentioned parameters is somewhat less than with $r = r_0$.

Curves presented in Figs. 2 and 3 indicate that σ_{rr} and u_t, depending on spatial coordinate r with fixed instants of time $t = 0.15 \cdot 10^{-3}$; $0.30 \cdot 10^{-3}$; $0.45 \cdot 10^{-3}$ (lines 1-3 in Fig. 2), with the exception of σ_{rr} for an elastoplastic material (solid line in Fig. 3) change mainly by a linear rule. In addition, stress σ_{rr} calculated for an elastoplastic material is least in absolute value. This picture occurs with a change in stresses σ_{rr}^* and $\sigma_{\phi\phi}^*$, and mass velocity u_t along the wave front in relation to time (Fig. 4).

With the aim of comparing the results of analytical and numerical methods, the method of characteristics was used in a computer to resolve the problem for spherical-wave propagation in soil for load $\sigma_0(t)$ obtained previously by the analytical reverse method (see Fig. 1, solid line). By comparing the results of calculations for distribution of material parameters along the SW front, it is noted that results of the characteristic method (broken-dotted line in Fig. 4) agree satisfactorily with the analytical solution of the problem.

Thus, if spherical SW front velocity is prescribed in the form of a uniformly decreasing function of time, then all of the material parameters in the disturbance region, including the loading profile at the cavity boundary, also take on a deteriorating function with time. Stress components and mass velocity for the material at the cavity boundary with the passage of time fell more rapidly than at the wave front. With an increase in coefficients α, α_2, β_1, and β_2 the stress components increase, and with an increase in Young's moduli E_1 and E_2 they decrease. However, the effect of E_2 on the distribution of material parameters is weaker than for E_1.

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LITERATURE CITED

1. Ya. L. Lunts, "Propagation of spherical elastoplastic waves," Prikl. Matem. i Mekhan., 13, No. 1 (1949).
2. N. V. Zvolinskii, "Emission of an elastic wave with a spherical explosion in soil," Prikl. Matem. i Mekhan., 24, No. 1 (1960).
3. S. S. Grigoryan and Ya. A. Pachevskii, "Effect of a strong underground explosion on a dense rock," Dokl. Akad. Nauk SSSR, 212, No. 2 (1973).
4. M. A. Sadovskii (ed.), Mechanical Effect of an Underground Explosion [in Russian], Moscow (1971).
5. P. F. Korotkov and B. M. Prosvirnina, "Numerical study of an explosion in an elasto-plastic material and some questions of modeling," Dokl. Akad. Nauk SSSR, 228, No. 1 (1976).
6. R. G. Yakupov, "Spherical explosive wave in solids," Fiz. Goren. Vzryv., 12, No. 5 (1976).
7. A. Ya. Sagomonyan and P. M. Garber, "Explosion of a spherical layer charge in a plastically compressible material," Vestn. MGU, Ser. 1, Matem. i Mekhan., No. 3 (1974).
8. Kh. A. Rakhmatulin, A. Ya. Sagomonyan, and N. A. Alekseev, Questions of Soil Dynamics [in Russian], MGU, Moscow (1964).
9. K. Atabaev and N. Mamadaliev, "Propagation of a unidimensional plastic wave in a material with linear and broken loads," Prikl. Matem. i Tekh. Fiz., No. 3 (1981).
10. N. Mamadaliev and A. I. Yusupov, "Propagation of unidimensional elastoplastic waves in soils," Prikl. Matem. i Tekh. Fiz., No. 5 (1982).
11. M. Mamadaliev and V. P. Molev, "Propagation of a two-dimensional plastic wave in a nonlinearly compressible half-space," Prikl. Matem. i Tekh. Fiz., No. 4 (1977).